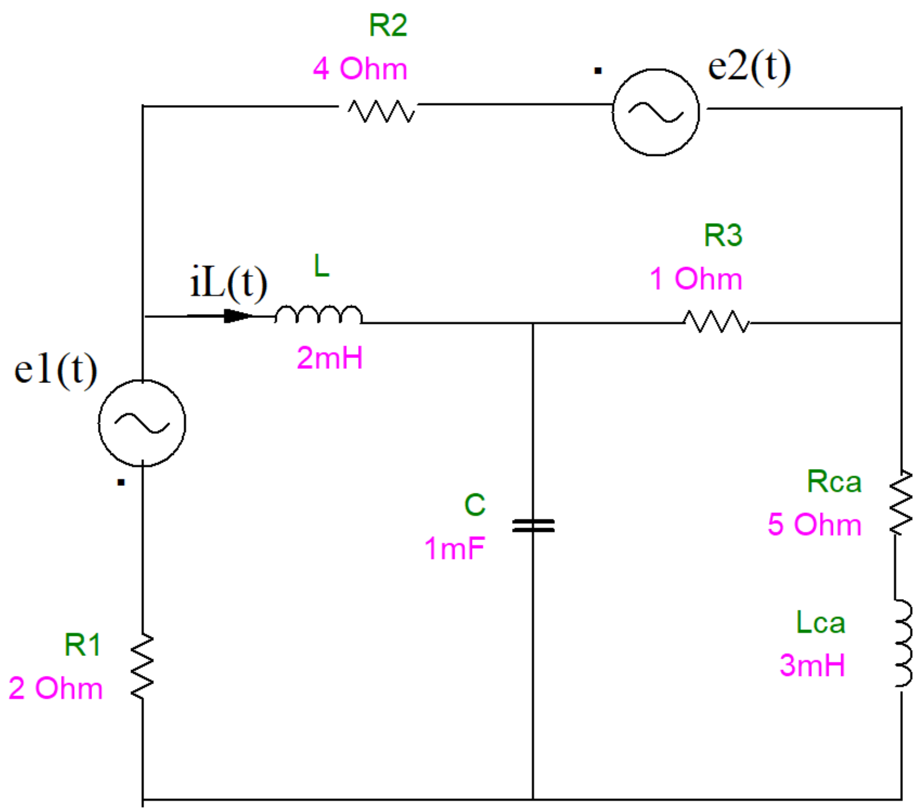
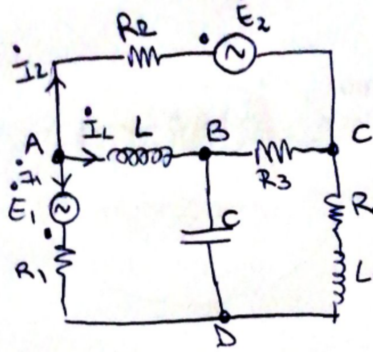
Dato il seguente circuito a regime, determinare la potenza attiva e reattiva che interessa il carico $Rca - Lca$ e determinare l'espressione temporale della corrente $i_L(t)$ che scorre sull'induttore L.

$e1(t) = \sqrt{2}\cos\left(wt + \frac{\pi}{4}\right)$; $e2(t) = 2\sqrt{2}\cos\left(wt + \frac{3\pi}{4}\right)$; f=50Hz;





#1

$$e_1(t) = \sqrt{2}\cos\left(\omega t + \frac{\pi}{4}\right) \Rightarrow \dot{E}_1 = \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

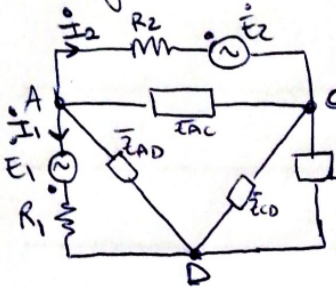
$$e_2(t) = 2\sqrt{2}\cos\left(\omega t + \frac{3\pi}{4}\right) \Rightarrow \dot{E}_2 = 2\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right) = -\sqrt{2} + j\sqrt{2} \ [V]$$

$$\bar{Z}_C = -\frac{j}{\omega C}$$

$$\bar{Z}_L = j\omega L$$

$$\bar{Z}_{CA} = R_{CA} + j\omega L_{CA}$$

Trasformazione stella (centro B) → triangolo



#2

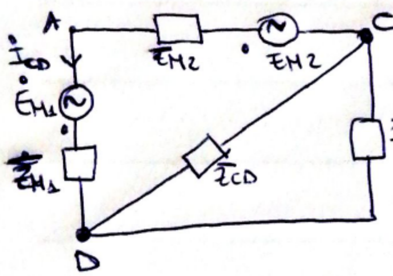
$$\bar{Z}_P = \left(\bar{Z}_C /\!/ \bar{Z}_L /\!/ R_3\right) = \frac{1}{\frac{1}{\bar{Z}_C} + \frac{1}{\bar{Z}_L} + \frac{1}{R_3}}$$

$$\bar{Z}_{AD} = \frac{\bar{Z}_L \cdot \bar{Z}_C}{\bar{Z}_P}$$

$$\bar{Z}_{AC} = \frac{R_3 \cdot \bar{Z}_L}{\bar{Z}_P}$$

$$\bar{Z}_{DC} = \frac{\bar{Z}_C \cdot R_3}{\bar{Z}_P}$$

Applico Millmann tra A-D e A-C:



#3

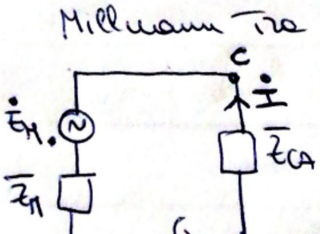
$$\dot{E}_{M1} = \frac{\frac{\dot{E}_1}{R_1}}{\frac{1}{R_1} + \frac{1}{\bar{Z}_{AD}}}$$

$$\bar{Z}_{M1} = \frac{1}{\frac{1}{R_1} + \frac{1}{\bar{Z}_{AD}}}$$

$$\dot{E}_{M2} = \frac{\frac{\dot{E}_2}{R_2}}{\frac{1}{R_2} + \frac{1}{\bar{Z}_{AC}}}$$

$$\bar{Z}_{M2} = \frac{1}{\frac{1}{R_2} + \frac{1}{\bar{Z}_{AC}}}$$

Millmann tra C e D:



#4

$$\dot{E}_M = \frac{\frac{\dot{E}_{M1} + \dot{E}_{M2}}{\bar{Z}_{M1} + \bar{Z}_{M2}}}{\frac{1}{(\bar{Z}_{M1} + \bar{Z}_{M2})} + \frac{1}{\bar{Z}_{CD}}}$$

$$\bar{Z}_M = \frac{1}{\frac{1}{\bar{Z}_{M1} + \bar{Z}_{M2}} + \frac{1}{\bar{Z}_{CD}}}$$

$$\dot{I} = \frac{\dot{E}_M}{\bar{Z}_M + \bar{Z}_{CA}}$$

Procedo con il calcolo della potenza attiva e reattiva sul carico $\bar{Z}_{CA}$:

$$\bar{S}_{CD} = \dot{V}_{CD} \cdot (-\check{I}) = (\bar{Z}_{CA} \cdot -\dot{I})(-\check{I}) = \underbrace{P_{CA}}_{\text{pot. attiva}} + j \underbrace{Q_{CA}}_{\text{pot. reatt.}}$$

Procediamo con il calcolo di $\dot{I}_L = -\dot{I}_1 - \dot{I}_2$ (legge al nodo A #1)

Conoscendo la $\dot{V}_{CD}$ dal #3 mi calcolo la $\dot{I}_{CD}$:

$$\dot{V}_{CD} = -\dot{E}_{M2} - \dot{E}_{M1} + \dot{I}_{CD}(\bar{Z}_{M1} + \bar{Z}_{M2}) \Rightarrow \dot{I}_{CD} = \frac{\dot{V}_{CD} + \dot{E}_{M1} + \dot{E}_{M2}}{\bar{Z}_{M1} + \bar{Z}_{M2}}$$

Conoscendo $\dot{I}_{CD}$ mi calcolo la $\dot{V}_{AC}$ e $\dot{V}_{AD}$ dal #3:

$$\dot{V}_{AC} = \dot{E}_{M2} - \dot{I}_{CD} \bar{Z}_{M2}$$

$$\dot{V}_{AD} = \dot{E}_{M1} + \dot{I}_{CD} \bar{Z}_{M1}$$

Dal #2 mi calcolo $\dot{I}_1$ e $\dot{I}_2$:

$$\dot{V}_{AD} = -\dot{E}_1 + \dot{I}_1 R_1 \Rightarrow \dot{I}_1 = \frac{\dot{V}_{AD} + \dot{E}_1}{R_1}$$

$$\dot{V}_{AC} = \dot{E}_2 + \dot{I}_2 R_2 \Rightarrow \dot{I}_2 = \frac{\dot{V}_{AC} - \dot{E}_2}{R_2}$$

quindi: $\dot{I}_L = -\dot{I}_1 - \dot{I}_2 = \mathrm{Re}\{I_L\} + j\,\mathrm{Im}\{I_L\}$

$$i_L(t) = \sqrt{2}\, |\dot{I}_L| \cos(\omega t + \varphi_{IL})$$

dove $|\dot{I}_L| = \sqrt{\mathrm{Re}\{I_L\}^2 + \mathrm{Im}\{I_L\}^2}$

$$\varphi_{IL} = \operatorname{arctg}\left(\frac{\mathrm{Im}\{I_L\}}{\mathrm{Re}\{I_L\}}\right)$$